COURSE DESCRIPTION: Systems of linear equations, matrices, and determinants; vector space and its subspaces, $\mathbb{R}^n$, coordinate system and bases; linear transformations; eigenvalues including complex eigenvalues, eigenvectors; inner product and orthogonality, orthogonal matrices; geometric and real-world applications.

COURSE GOALS: To introduce to the student the concepts, methods and applications of topics in linear algebra necessary for further study in sciences and engineering; to present key ideas and concepts from a variety of perspectives, and connections to other subjects; to develop student's mathematical thinking and problem solving ability; and to facilitate student’s progression from a procedural/computational understanding of mathematics to a broader understanding encompassing logical reasoning, generalization, abstraction, and formal proof.

GENERAL EDUCATION GOALS: Critical Thinking and Quantitative Literacy

SPECIAL COURSE REQUIREMENTS: None

TEXTBOOK, MANUALS, REFERENCES, AND OTHER REQUIRED MATERIALS


- A graphing calculator is recommended. The TI-89, TI-92, TI-Nspire CAS, and other Computer Algebra Systems (CAS) are never allowed during proctored assessments.

GENERAL INSTRUCTIONAL METHODS
Classroom lecture, discussion, recitation, and/or problem solving explorations supplemented by visual and/or computer aids.

ASSESSMENT: Columbus State Community College is committed to assessment (measurement) of student achievement of academic outcomes. This process addresses the issues of what you need to learn in your program of study and if you are learning what you need to
learn. The assessment program at Columbus State has four specific and interrelated purposes: (1) to improve student academic achievements; (2) to improve teaching strategies; (3) to document successes and identify opportunities for program improvement; (4) to provide evidence for institutional effectiveness. In class you are assessed and graded on your achievement of the outcomes for this course. You may also be required to participate in broader assessment activities.

STANDARDS AND METHODS FOR EVALUATION
The final examination must account for between 25% and 35% (inclusive) of the course grade. The remainder of the course grade is to be determined by the instructor, subject to the following departmental policies:

- NO credit is to be awarded for attendance and/or class participation.
- NO credit is to be awarded for assignments that are only checked for completion rather than graded for correctness.
- Routine homework (e.g. MyMathLab and textbook exercises) should account for no more than 15% of the course grade. Group work and special projects, if utilized, should account for no more than 10% of the course grade. At least 75% of the course grade must be based on proctored, closed book quizzes, tests, and/or final exam. (There may be situations where exceptions to these caps are appropriate. Please discuss such cases with the Lead Instructor(s) of the course prior to straying from these guidelines.)
- Eliminate extra credit assignments, or limit them to no more than 2% of the course grade.

GRADING SCALE
Letter grades for the course will be awarded using a 90% - 80% - 70% - 60% scale.

ATTENDANCE POLICY: To be determined by the instructor.

MAKEUP POLICY: To be determined by the instructor.

UNITS OF INSTRUCTION
Please provide a weekly course schedule indicating the Units of Instruction, learning objectives/goals, assigned readings, assignments, and exams.

Week 1
- Unit of Instruction: Linear Equations in Linear Algebra (Systems of linear Equations; Row Reduction and Echelon Forms; Vector Equations) - Student Learning Outcomes:
  1. Represent systems of linear equations in matrix form.
  2. Solve systems of linear equations.
  3. Demonstrate the ability to perform elementary row operations (using Gaussian-Jordan elimination method) to reduce a matrix to: i. Echelon form, ii. Reduced echelon form
4. Understand algebraic and geometric representations of vectors in $\mathbb{R}^n$ and their operations, including addition, scalar multiplication.

5. Determine if a given vector is a linear combination of other given vectors.

6. Give a geometric interpretation for the Span of a set of vectors.

**- Assigned Reading:** 1.1, 1.2, 1.3

**- Assessment Methods:** Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.

**Week 2**

**- Unit of Instruction:** Linear Equations in Linear Algebra (The Matrix Equation $Ax = b$; Solution Sets of Linear Systems; Linear Independence) - **Student Learning Outcomes:**

1. Compute the product of a matrix and a vector.

2. Determine if a given set of $n$ vectors spans $\mathbb{R}^n$.

3. Employ matrix reduction techniques to solve systems of linear equations including the identification of inconsistent and dependent systems.

4. Interpret geometrically existence and uniqueness of solutions.

5. Identify independent and dependent sets of vectors.

**- Assigned Reading:** 1.4, 1.5, 1.7

**- Assessment Methods:** Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.

**Week 3**

**- Unit of Instruction:** Linear Equations in Linear Algebra (Introduction to linear Transformations; The Matrix of a Linear Transformation; Linear Models in Business, Science, and Engineering)

**- Student Learning Outcomes:**

1. Find the image of a given vector under a given transformation.

2. Interpret geometrically the effect certain linear transformations have on a vector.

3. Express a given linear transformation as a matrix.

4. Find the standard matrix of a given linear transformation.

5. Determine if a linear transformation is one-to-one.

6. Set up difference equations describing population movement.
- **Assigned Reading:** 1.8, 1.9, 1.10
- **Assessment Methods:** Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.

**Week 4**
- **Unit of Instruction:** Matrix Algebra (Matrix Operations, The Inverse of a Matrix) - **Student Learning Outcomes:**
  1. Compute sums, scalar products, and differences using matrices.
  2. Multiply matrices, and understand associativity and noncommutativity of matrix multiplication.
  3. Interpret a matrix product as a composition of linear transformations.
  4. Compute the transpose of a matrix.
  5. Given an invertible matrix, compute its inverse.
  6. Solve systems of linear equations using the inverse of the coefficient matrix when possible.
- **Assigned Reading:** 2.1, 2.2
- **Assessment Methods:** Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.

**Week 5**
- **Unit of Instruction:** Matrix Algebra (Characterizations of Invertible Matrices); Determinants (Introduction to Determinants) - **Student Learning Outcomes:**
  1. Determine if a given $n \times n$ matrix is invertible.
  2. Compute the determinant of an $n \times n$ matrix using a cofactor expansion.
  3. Compute the determinant of a triangular matrix.
- **Assigned Reading:** 2.3, 3.1, 3.2
- **Assessment Methods:** Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.

**Week 6**
- **Unit of Instruction:** Determinants (Properties of Determinants); Vector Spaces (Vector Spaces and Subspaces)
  - **Student Learning Outcomes:**
    1. Identify various properties of determinants.
    2. Understand algebraically and geometrically the determinant of product.
    3. Combine methods of row reduction with cofactor expansion to compute determinants.
4. Use determinants to decide if a matrix is invertible.

5. Interpret determinants as volumes and use this interpretation to solve related problems.

6. Understand and identify axiomatically an abstract vector space.

7. Determine if a given vector space is a subset of another given vector space.

- **Assigned Reading:** 3.2, 4.1  
- **Assessment Methods:** Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.

**Week 7**

- **Unit of Instruction:** Vector Spaces (Vector Spaces and Subspaces, Null Spaces, Column Spaces, and Linear Transformations)  
- **Student Learning Outcomes:**

1. Determine if a given vector is in the space spanned by a set of vectors.

2. Understand relationship between kernel and range (image) of a linear transformation and nullity and rank of a matrix.

3. Determine if a given vector is in the null space and/or column space of a given matrix.

4. Find a spanning set for the null space of a given matrix.

5. Determine if a given set \( W \) forms a vector space.

- **Assigned Reading:** 4.1, 4.2  
- **Assessment Methods:** Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.

**Week 8**

- **Unit of Instruction:** Vector Spaces (Linearly Independent Sets, Bases; Coordinate Systems)  
- **Student Learning Outcomes:**

1. Determine if a given set of vectors are linearly independent.

2. Find a basis for the space spanned by a given set of vectors.

3. Given a matrix \( A \), find a basis for its column space.

4. Given a basis \( B \) and coordinate vector \( \mathbf{x} \), find the vector \( \mathbf{x} \).

5. Given a basis \( B \) and vector \( \mathbf{x} \), find the coordinate vector \( \mathbf{y} \).

- **Assigned Reading:** 4.3, 4.4  
- **Assessment Methods:** Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.
Week 9
- **Unit of Instruction:** Vector Spaces (Coordinate Systems; The Dimension of a Vector Space) -
- **Student Learning Outcomes:**
  1. Find the change-of-coordinates matrix from a given basis to the standard basis.
  2. State the dimension of a given vectors space.
  3. Find the dimension of a space spanned by given vectors.
- **Assigned Reading:** 4.4, 4.5
- **Assessment Methods:** Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.

Week 10
- **Unit of Instruction:** Vector Spaces (Rank; Change of Basis) -
- **Student Learning Outcomes:**
  1. Given a matrix $A$, find bases for the row space, the column space, determine the rank and the null space.
  2. Prove and apply the theorems concerning the rank and the relationship between rank and nullity.
  3. Find the change-of-coordinates matrix.
  4. Given a situation, find the stochastic matrix, find the initial state vector, and apply this information in predicting.
- **Assigned Reading:** 4.5, 4.6, 4.7, 4.9
- **Assessment Methods:** Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.

Week 11
- **Unit of Instruction:** Eigenvalues and Eigenvectors (Eigenvectors and Eigenvalues; The Characteristic Equation)
- **Student Learning Outcomes:**
  1. Determine if a given number is an eigenvalue of a given matrix. If so, find its corresponding eigenvector.
  2. Determine if a given vector is an eigenvector of a given matrix. If so, find its corresponding eigenvalue.
  3. Find the characteristic polynomial and the eigenvalues of a given matrix.
- **Assigned Reading:** 5.1, 5.2, 5.3
- **Assessment Methods:** Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.
Week 12
- **Unit of Instruction:** Eigenvalues and Eigenvectors (Diagonalization; Eigenvectors and Linear Transformations; Complex Eigenvalues; Discrete Dynamical Systems) - **Student Learning Outcomes:**
  1. Understand diagonalizable matrix.
  2. Use eigenspace to diagonalize a matrix.
  3. Represent a linear transformation by a matrix relative to given bases.
  4. Determine the complex eigenvalues of a given matrix, and a basis for the corresponding eigenspace.
  5. Determine the long-term behavior of various dynamical systems, and for various values of the parameter $p$.
- **Assigned Reading:** 5.3, 5.4, 5.5, 5.6
- **Assessment Methods:** Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.

Week 13
- **Unit of Instruction:** Orthogonality and Least Squares (Inner product; Length, and Orthogonality)
- **Student Learning Outcomes:**
  1. Compute the inner product (dot product) of two vectors.
  2. Compute the distance between two vectors.
  3. Compute the angle between two vectors and determine if two vectors are orthogonal.
  4. Understand the orthogonal complement.
- **Assigned Reading:** 6.1, 6.2
- **Assessment Methods:** Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.

Week 14
- **Unit of Instruction:** Orthogonality and Least Squares (Orthogonal Sets; Orthogonal Projections)
- **Student Learning Outcomes:**
  1. Determine if a given set of vectors are orthogonal.
  2. Understand and compute orthogonal projection.
3. Show that a given set of vectors form an orthogonal basis and an orthonormal basis for the vector space $\mathbb{R}^2$ or $\mathbb{R}^3$.

4. Decompose a given vector $y$ as the sum of a vector parallel to another vector $u$ and one orthogonal to $u$.

5. Understand orthogonal matrix.

6. Find the orthogonal projection of a vector $y$ onto the subspace spanned by two orthogonal vectors $u_1$ and $u_2$.

- Assigned Reading: 6.2, 6.3
- Assessment Methods: Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.

**Week 15**

- Unit of Instruction: Orthogonality and Least Squares (The Gram-Schmidt Process; Least-Squares Problems)

- Student Learning Outcomes:
  1. Apply the Gram-Schmidt Process.
  2. Find a least-squares solution to the equation $Ax = b$.
  3. Determine the equation of the least-squares polynomial of a given degree that best fits given data points.

- Assigned Reading: 6.4, 6.5, 6.6
- Assessment Methods: Daily questioning, graded homework assignments, quizzes and/or tests. Out of class assignments allowing for greater computational and conceptual complexity.
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